UDC 536.3

ON THE DETERMINATION OF THE RADIATION SHAPE FACTOR OF A SYSTEM OF TWO COAXIAL CYLINDERS

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Calculation of radiation shape factors is presented for a system of two coaxial cylinders of equal height. The shape factor for self-irradiation of the outer cylinder is plotted versus the ratio of the diameters of the inner and outer cylinders for various ratios of the outer cylinder diameter to its height. Similar plots are also presented for the shape factors for radiation from the outer cylinder to the inner one and from the inner cylinder to the bases. The maximum error in the predicted shape factors does not exceed 7 percent.

In order to calculate the radiative heat transfer between surfaces arranged in an arbitrary geometrical order one must know the radiation shape factors, defined as the fraction of the radiative flux emitted by one body which is intercepted by the other body.

In accordance with Lambert's law [1] the shape factor of body 1, irradiating body 2, is

$$\varphi_{12} = \frac{1}{\pi S} \int\limits_S dS \int\limits_\sigma \frac{\cos\varphi_S \cos\varphi_\sigma}{r^2} d\sigma . \tag{1}$$

Equation (1) has been used to calculate the shape factors for various configurations which appear in laboratory and industrial practice, and the results have been represented in the form of formulas and charts [1, 2]. In the present work we shall calculate the shape factors for a system of two coaxial cylindrical surfaces, which has not yet been treated in heat-transfer literature. The radiative heat transfer is assumed to take place between the outer surface of the inner cylinder and the inner surface of the outer cylinder.

In general, this system can be characterized by five shape factors: φ_{ob}, φ_{oi}, φ_{oo}, which are associated with the outer cylinder and denote the fractions of the radiative flux emitted by the outer cylinder which are intercepted by the bases, the inner cylinder, and the outer cylinder (self-irradiation), respectively, and φ_{ib}, φ_{io}, which denote the fractions of the radiative flux emitted by the inner cylinder which are intercepted by the bases and the outer cylinder, respectively.

Only two of these five factors can be chosen arbitrarily. The other three factors are then determined from the equations:

$$\varphi_{ob} + \varphi_{oi} + \varphi_{oo} = 1, \tag{2}$$

$$\varphi_{ib} + \varphi_{io} = 1, \tag{3}$$

$$\varphi_{io} S_i = \varphi_{oi} S_o. \tag{4}$$

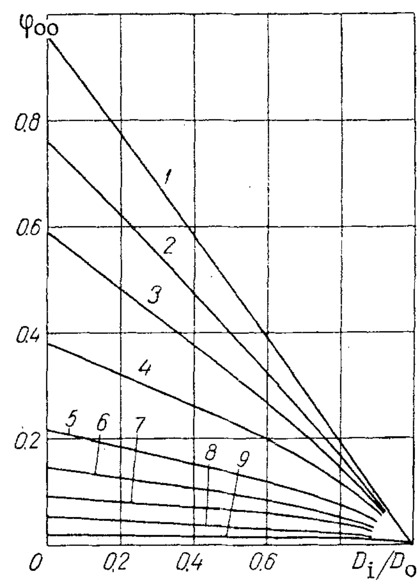

Fig. 1. Shape factor for self-irradiation φ_{oo} as a function of the ratio of the diameters of the inner and outer cylinder β/α: 1) $D_o/l = 0.1$; 2) 0.5; 3) 1.0; 4) 2.0; 5) 4.0; 6) 6.0; 7) 10; 8) 20; 9) 40.

Here we shall choose φ_{oo} and φ_{oi} as the two independent factor.

Using simple geometric constructions, Eq. (1) yields the following expressions for φ_{oo} and φ_{oi}:

$$\varphi_{oo} = \frac{\alpha R_i^2}{\pi^2} \int\limits_0^l dZ_\sigma \int\limits_0^l dZ_S \int\limits_0^{2\pi} d\varphi_\sigma \int\limits_0^{2\arccos\beta/\alpha} \frac{(1-\cos\varphi_S)^2 \, d\varphi_S}{[2R_o^2(1-\cos\varphi_S)+(Z_S-Z_\sigma)^2]^2} , \tag{5}$$

$$\varphi_{oi} = \frac{\beta}{\pi^2} \times \int\limits_0^l dZ_\sigma \int\limits_0^l dZ_S \int\limits_0^{2\pi} d\varphi_\sigma \int\limits_0^{\arccos\beta/\alpha} \frac{(R_o - R_i\cos\varphi_S)(R_o\cos\varphi_S - R_i)\, d\varphi_S}{[R_o^2 + R_i^2 - 2R_oR_i\cos\varphi_S + (Z_S - Z_\sigma)^2]^2} . \tag{6}$$

After some simple transformations, Eq. (5) reduces to the form

$$\varphi_{oo} = 1 - \frac{2}{\pi}\,\beta/\alpha\,\mathrm{arc\,tg}\,\frac{1}{\sqrt{1-(\beta/\alpha)^2}} + \frac{1}{\pi\alpha}\,\mathrm{arc\,sin}\,\sqrt{1-(\beta/\alpha)^2} - \frac{\sqrt{1+4\alpha^2}}{\pi\alpha}\,\mathrm{arc\,tg}\,[\sqrt{1+4\alpha^2} \times \mathrm{tg\,arc\,sin}\,\sqrt{1-(\beta/\alpha)^2}]. \tag{7}$$

In the special case $\beta = 0$ this reduces to the simpler form

$$\varphi'_{oo} = 1 + \frac{1}{2\alpha} - \sqrt{1+\left(\frac{1}{2\alpha}\right)^2}. \tag{8}$$

which coincides with the expression for the shape factor of self-irradiation of a single cylinder given in [2].

Figure 1 represents φ_{oo} as a function of the ratio of the diameters of the inner and outer cylinders β/α according to (7). It can be seen that φ_{oo} increases with increasing height and diameter of the outer cylinder (l and D_o), with the inner diameter D_i being constant.

Equation (6) for the shape factor φ_{oi} can be rephrased as

$$\varphi_{oi} = \frac{2}{\pi}\,\beta\,(\alpha^2+\beta^2)\int\limits_0^{\mathrm{arc\,cos}\,\beta/\alpha} I(\varphi_S)\cos\varphi_S\,d\varphi_S - \frac{2}{\pi}\,\alpha\beta^2\int\limits_0^{\mathrm{arc\,cos}\,\beta/\alpha} I(\varphi_S)\,[1+\cos^2\varphi_S]\,d\varphi_S, \tag{9}$$

where

$$I(\varphi_S) = \frac{1}{(\alpha^2+\beta^2-2\alpha\beta\cos\varphi_S)^{3/2}}\,\mathrm{arc\,tg}\left(\frac{1}{\alpha^2+\beta^2-2\alpha\beta\cos\varphi_S}\right)^{1/2}. \tag{10}$$

Further attempts to reduce Eq. (9) to elementary or tabulated integrals were unsuccessful. Thus Eq. (9) was integrated numerically with a prescribed maximum error $\varepsilon_{max} \leq 7\%$.

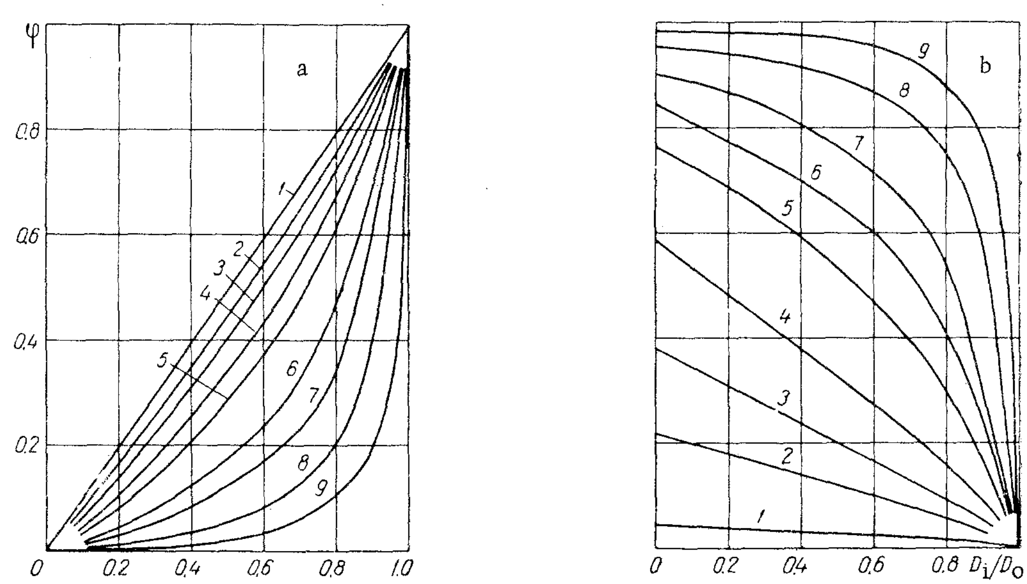

Fig. 2. The shape factors φ_{oi} (a) and φ_{ib} (b) as functions of the ratio β/α for various values of D_o/l. 1)-9) Cf. Fig. 1

Figure 2a represents φ_{oi} as a function of β/α for various values of D_o/l. It can be seen that the shape factor φ_{oi} increases with decreasing outer diameter D_o and with increasing height l, with the inner diameter D_i being constant.

The data of Figs. 1 and 2, together with Eqs. (2), (3) and (4), yield the three remaining shape factors φ_{ob}, φ_{ib}, and φ_{io} with the same maximum error ε_{max}.

Thus, Fig. 2b represents the shape factor φ_{ib} as a function of the ratio of the diameters of the inner and outer cylinder β/α for several values of D_o/l.

NOTATION

dS and dσ – area elements of surfaces 1 and 2, respectively; S – area of surface 1; r – distance between the centers of area elements dS and dσ; φ_S and φ_σ – angles between the line r and the normals to surfaces dS and dσ, respectively; S_i and S_o – surface areas of the inner and outer cylinders; $\beta = R_i/l$ and $\alpha = R_o/l$ – ratios of the radii of the inner and outer cylinders and the cylinder height; Z_σ and Z_S – coordinates of the area elements dσ and dS along the axis; D_o and D_i – diameters of the outer and inner cylinder, respectively.

REFERENCES

1. A. G. Blokh, Fundamentals of Radiative Heat Transfer [in Russian], Gosenergoizdat, 1962.
2. A. I. Ansel'm, Tekhnika zavoda "Svetlana," vol. 3, no. 13, Gosenergoizdat, 1932.

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